Water abundance prediction method of weathered bedrock based on improved AHP and the entropy weight method

Enke Hou[1], Zhuochen Ji[1], Xiaoyang Che[1], Jianwen Wang[2], Lijun Gao[2], Shixiong Tian[2], Fan Yang[3]

[1]*School of Geology and Environment, Xi'an University of Science and Technology, Xi'an 710054, China;*
[2]*Shenmu Ningtiaota Coal Mining Company Ltd, Shaanxi Coal and Chemical Industry Company Ltd, Yulin 719300, China;*
[3]*Shenmu Zhangjiamao Coal Mining Company Ltd, Shaanxi Coal and Chemical Industry Company Ltd, Yulin 719316, China*

Abstract

The Jurassic weathered bedrock aquifer in Northwest China is the main aquifer for filling water in coal mines. Many water inrush disasters in the coal mine are related to the aquifer. The south wing of Ningtiaota coalfield was taken as an example to analyze the water aboundance of weathered bedrock and its controlling factors. The water-rich zoning of weathered bedrock aquifer is carried out by using AHP and entropy weight method to calculate the weights separately and coupled with each other. The results indicated that coupled of improved AHP and the entropy weight method can be used to predicte the water-richness of weathered bedrock accurately.

Keywords: AHP, the entropy weight method, weathered bedrock,water abundance, Ningtiaota coalfield

Introduction

Northwest China is rich in coal resources and is the main coal production area in China at present and in the future(Chen and Jiang 2007). In recent years, many water inrush disasters of Jurassic weathered bedrock occurred in the mining of Jurassic coal resources in Northwest China, which made people gradually realize the importance of mastering the water richness of Jurassic weathered bedrock in mine water prevention. However, the water aboundance of Jurassic weathered bedrock is very uneven in space. How to evaluate and predict the water-rich of it has become an urgent problem to be solved. The evaluation and prediction of aquifer's water-enrichment is an important basic work in mine water disaster prevention and control (Xi et al. 2015). Many scholars have studied it and put forward a multi-factor comprehensive analysis method (Dai et al. 2016; Wu et al. 2011; Hou et al. 2016; Wang et al. 2014), which is on the basis of analyzing the influencing factors of aquifer's water-enrichment, the quantitative evaluation and prediction model of aquifer's water-enrichment is established. It is very important to determine the weight of each factor when a model of aquifer water-rich evaluation and prediction is established. Because different weights have different prediction results for water abundance, and it is often determined by Analytic Hierarchy Process (AHP). However, most of the weights of water-rich influencing factors in AHP are determined by experts according to the 1-9 scale method created by T.L.SAATY, and the weights of each factor are not make full use of the measured data and the calculated weights are subjective (Guo et al. 2008). Zhang et al. (2006) proposed an improved method for determining the weight of AHP indicators. In addition to considering the scoring of experts, the method also considers the standard deviation of the measured data of each index, which improves the application effect to a certain extent. In order to overcome the subjectivity of AHP computing weights and predicted the heterogeneity of the aquifer, the entropy weight method was adopted by Ma et al. (2011). The advantage of this method is an objective evaluation method for determining the weight based on the measured data of each index, but the disadvantage is that the expert's experience is not considered when determining the weight of each index. Therefore, there are certain limitations to using either method alone.The south wing of Ningtiaota coalfield in North Shaanxi is used as example, discussing the prediction method of weathered bedrock water abundance by coupling the improved AHP and entropy weight method to determine the index weight. The coupling is through reasonable mathematical methods, which considering both expert experience and measured data characteristics, so that the prediction results of water richness evaluation are more accurately.

General Situation of the Study Area

The Jurassic coalfield is one of the main coal producing areas between Shanxi, Shannxi and Inner Mongolia of north China, which coal resources account for about 14% of the national total. The study area is located in south wing of Ningtiaota coalfield in Shenmu City, Shannxi Province. The overlying aquifer is porous phreatic water of Quaternary, fissure confined water in weathered bedrock of Zhiluo Formation and fissure confined water in Yan'an Formation. The fissure confined water in weathered bedrock of Zhiluo Formation is widely distributed in the study area, with an average thickness of 26.62 m. The unit inflow of borehole is 0.1183 L/s.m, which is the main water-filling aquifer and poses more threat to mine safety production.

Controlling factors of weathered bedrock water abundance

In order to evaluate water enrichment more effectively, author analyzed 41 pumping tests data of weathered bedrock aquifer in the study area. It was found that the water enrichment of weathered bedrock was mainly controlled by 4 factors:

1. Top-level index

According to the analysis of pumping tests data, there is a certain correlation between the water abundance of weathered bedrock and its top-level. For example, the unit inflow of boreholes BK42 and K3-1 are 0.1051L/s.m and 0.1220L/s.m. The corresponding top-level are 1180.40m and 1218.32. In order to clearly express this correlation, the elevation of weathered bedrock is graded according to the height of 15 m. The larger the value, the stronger the water richness.

2. Lithologic association index

According to the data of pumping boreholes, use the contribution of the lithology combination to the water-richness, it is quantified (Table 2). The larger the value, the stronger the water-richness.

3. Weathered index

The stronger the weathering degree is, the better the water-rich is. Quantify rock weathering by category (Table 3). The weathering influence index w is constructed to describe the influence of weathering degree

***Table 1** Assignment to weathered bedrock top elevation level*

Elevation m	>1240	1225–1240	1210–1225	1195–1210	<1195
Quantized value	1	2	3	4	5

***Table 2** Lithologic grade assignment of weathered bedrock*

Lithology category	Mudstone	Sandy mudstone	Siltstone	Fine sandstone	Medium sand	Coarse sandstone	Conglomerate
Quantized value	1	1.5	2	3	4	5	6

***Table 3** Assignment to weathering degree of bedrock*

Weathering degree	Weak	Medium	Strong
Quantized value	1	2	3

and thickness on water abundance. The larger the value, the stronger the water-rich.

$$w = \sum w_i h_i \Big/ \sum h_i \qquad (2)$$

In this formula, w_i is weathered degree of different lithologic rocks, h_i is thickness of wreathered bedrock with different lithology.

4 Core rate index

The lower the value and the better water richness. The core rate of weathered bedrock in each borehole is calculated by constructing core recovery index c, which reflects the water-rich of weathered bedrock from the side. The smaller the value, the stronger the water richness.

$$c = \sum c_i h_i \Big/ \sum h_i \qquad (3)$$

Where c_i is rock formation rate; h_i is strata thickness.

In conclusion, water abundance of weathered bedrock can be predicted by four indexs: top-level index, lithologic association index, weathered index and core rate index. The hierarchical structure model is shown in Figure 1.

Method

Improved AHP indicator weight determination method

The specific content of the improved analytic hierarchy process is as follows: for each of n index, the standard deviation of samples is calculated separately, then the standard deviation of samples is compared with each other, and the values of other elements of the judgment matrix can be obtained in turn (Huang and Zheng 2003).

The sample standard deviation $S(i)$ of each evaluation index can be used to reflect the influence degree of each evaluation index on aquifer water-rich, and to construct a judgment matrix $B_{n\times n}$, The calculation formula of its internal value b_{ij} is as follows:

$$b_{ij} = \begin{cases} \dfrac{S(i)-S(j)}{S_{\max}-S_{\min}}(b_m-1)+1, S(i)\geq S(j) \\ 1\Big/\left[\dfrac{S(j)-S(i)}{S_{\max}-S_{\min}}(b_m-1)+1\right], S(i)<S(j) \end{cases}$$

$$b_m = \min\{9, \mathrm{int}[S_{\max}/S_{\min}+0.5]\}$$

In this formula, $S_{\max}$ and $S_{\min}$ are the maximum and minimum values of $S(i)$ respectively; Relative importance parameter b_m, min and int are the smallest and integral functions respectively. Finding the eigenvector u_j corresponding to the maximum eigenvalue $\lambda_{\max}$ according to the matrix.

$$M_i = \prod_{j=1}^{n} b_{ij} \qquad (i=1,2,3,\mathrm{L}\ \ ,n) \qquad (5)$$

$$u_j = \overline{u_i}\Big/\left(\sum_{i=1}^{n}\overline{u_i}\right) \qquad (6)$$

Because of the complexity of multi-order judgment matrix and some values in the judgment matrix may be inconsistent. It is necessary to check the consistency of

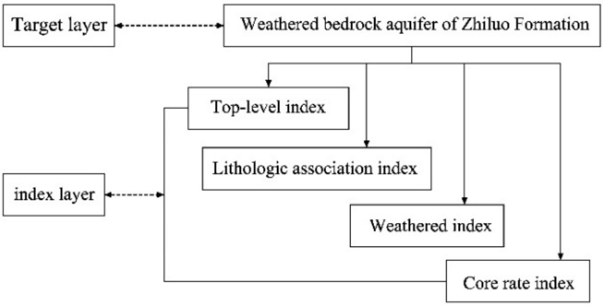

Fig.1 *Hierarchy model for predicting water-richness of weathered bedrock aquifer*

judgment matrix to ensure the consistency of final evaluation, the consistency test calculation formula is as follows:

$$CI = \left(\lambda_{\max} - n\right)/n - 1 \tag{7}$$

$$RI = CI/CR \tag{8}$$

In this formula, u_j is the weight of improved AHP calculation, n is order of judgment matrix , $\lambda_{\max}$ is maximum eigenvalue, when CR <0.1, the judgment matrix satisfies the consistency test, otherwise it needs to be adjusted.

Entropy weight method index weight determination method

Entropy is a concept in thermodynamics that was introduced into information theory by C. E. Shannon to measure the amount of information. For a system the more ordered, the lower the information entropy; the more disordered, the higher the information entropy (Deng et al. 2017). The entropy is an objective way to determine the weight. It is based on the degree of variation of indicators, which can eliminate the subjective interference of people as much as possible (Kong et al. 2018). As described below:

1. Raw data was collected to carry out the standardized treatment.

The original data matrix X is consisted by n evaluation objects and m evaluation indexes:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \tag{9}$$

Standardize X to get Y:

$$Y = \left(y_{ij}\right)_{n \times m} \tag{10}$$

In this formula, y_{ij} is the standard value of the j evaluation index on the i evaluation object. The evaluation index can be generally divided into positive index y_{ij} (the larger the index value, the better) and negative index y_{ij}' (the smaller the index value, the better).

$$y_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \tag{11}$$

$$y_{ij}' = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}} \tag{12}$$

2. Entropy of Evaluation Index

In the evaluation questions of n evaluation targets and m evaluation indicators, the entropy of the j index is defined as:

$$H_j = -k \sum_{i=1}^{n} f_{ij} \ln f_{ij}, \quad \left(j = 1, 2, 3, \cdots, m\right) \tag{13}$$

In this formula, $f_{ij} = y_{ij} / \sum_{i=1}^{n} y_{ij}$, $k = 1/\ln n$,

The smaller entropy value, the more useful information the index provides to decision makers.

3. Evaluation index entropy weight

The entropy weight of the *j* index is defined as:

$$v_j = 1 - H_j \Big/ m - \sum_{j=1}^{m} H_j \tag{14}$$

In this formula,

$$0 \le v_j \le 1, \quad \sum^{m} v_j = 1,$$

v_j is the weight calculated by the entropy weight method.

Method for determining index weights for coupling method

In this paper, the least square method of optimization method is used to couple improved AHP and the entropy weight to form a subjective and objective weighting method.

$$\min F(w) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left\{ \left[\left(u_j - w_j\right) z_{ij} \right]^2 + \left[\left(v_j - w_j\right) z_{ij} \right] \right. \tag{15}$$

The constraint condition is as follows:

$$\sum_{j=1}^{m} w_j = 1, \quad w_j \ge 0\left(j = 1, 2 \cdots \cdots m\right)$$

Represented by a matrix:

$$\begin{bmatrix} A & e \\ e^T & 0 \end{bmatrix} \times \begin{bmatrix} w_j \\ \lambda \end{bmatrix} = \begin{bmatrix} B \\ 1 \end{bmatrix} \tag{16}$$

In this formula, A is a diagonal array. e, W and B are m rows and one column vector.

$$A = diag\left[\sum_{i=1}^{n} z_{i1}^{2}, \sum_{i=1}^{n} z_{i2}^{2}, \cdots, \sum_{i=1}^{n} z_{im}^{2}\right] \quad (17)$$

$$e = [1, 1, \cdots, 1]^T \quad w_j = [w_1, w_2, \cdots, w_m]^T$$

$$B = \left[\sum_{i=1}^{n} \frac{1}{2}(u_1 + v_1) z_{i1}^{2}, \sum_{i=1}^{n} \frac{1}{2}(u_2 + v_2) z_{i2}^{2}, \cdots, \sum_{i=1}^{n} \frac{1}{2}(u_m + v_m) z_{im}^{2}\right]^T \quad (18)$$

Solve the matrix equation above and get w_j:

$$w_j = A^{-1} \times \left[B + \frac{1 - e^T A^{-1} B}{e^T A^{-1} e} \times e\right] \quad (19)$$

Where w_j is the weight of the improved AHP and entropy weight method coupling calculation, z_{ij} is a standardized data matrix with m evaluation indicators and n evaluation objects.

Prediction of weathered bedrock water abundance in study area

Water richness index weight calculation

According to the data of 172 boreholes in the study area, the weight of water-rich prediction indicators are calculated by improved AHP, entropy weight method and the coupling method.

1. Index Weight Calculation Based on Improved AHP

The judgment matrix $B_{4\times4}$ calculated according to the formula 4 is as follows:

$$B_{4\times4} = \begin{bmatrix} 1 & 4.41 & 5.23 & 9 \\ 0.23 & 1 & 1.81 & 5.58 \\ 0.19 & 0.55 & 1 & 4.77 \\ 0.11 & 0.18 & 0.21 & 1 \end{bmatrix}$$

The judgment matrix is a 4×4 matrix, which is calculated by formulas (7) and (8). The maximum eigenvalue of the judgment matrix is λ_{max} =4.1513, CR=0.0348<0.1. Therefore, the judgment matrix satisfies the consistency test.

The weight of the index layer u_j is obtained by calculation: The weight u_1 of top-level index is 0.6197, the weight of lithologic association index u_2 is 0.2016, the weight of weathered index u_3 is 0.1372, and core rate index u_4 is 0.0415(Table 4).

2. Index Weight Calculation Based on Entropy Weight

The matrix normalized by Equation 10 as follows:

$$Y = \begin{bmatrix} 0.2500 & 0.2950 & 0.4850 & 0.8939 \\ 0.0000 & 0.5000 & 0.6787 & 0.2641 \\ \vdots & \vdots & \vdots & \vdots \\ 0.2500 & 0.5000 & 0.5826 & 0.2643 \end{bmatrix}$$

The normalized matrix is calculated by using formulas (13) and (14) in Matlab, and the weight v_j of each index is obtained: The weight v_1 of top-level index is 0.4136, the weight of lithologic association index v_2 is 0.1779, the weight of weathered index v_3 is 0.2614, and core rate index v_4 is 0.1471(Table 4).

3. Index Weight Calculation Based on the Coupling Method

The improved AHP and the entropy weight method are coupled by the least squares method. From equation (17), A is a 4×4 diagonal matrix, as follows:

$$A = \begin{bmatrix} 51.6875 & 0 & 0 & 0 \\ 0 & 74.5079 & 0 & 0 \\ 0 & 0 & 49.5930 & 0 \\ 0 & 0 & 0 & 51.0274 \end{bmatrix}$$

In matlab, formulas (18) and (19) are used to calculate the optimal weight w_j of the index layer: The weight w_1 of top-level index is 0.5166, the weight of lithologic association index w_2 is 0.1898, the weight of weathered index w_3 is 0.1993, and core rate index w_4 is 0.0943(Table 4).

Table 4 Weighted prediction index of weathered bedrock calculated by different methods

indicators	top-level index of the weathered bedrock	lithologic association index	weathered index	core rate index
AHP weight (u_j)	0.6197	0.2016	0.1372	0.0415
The entropy weight (v_j)	0.4136	0.1779	0.2614	0.1471
Coupling weight (w_j)	0.5166	0.1898	0.1993	0.0943

Evaluation and Verification of Water Abundance

According to the standardized data rasterized in Surfer, the grid maps of each index are imported into ArcGIS, then the raster calculator is used. Combined with the improved AHP, the entropy weight method and the weight calculated by coupling, the overlay analysis is carried out respectively. Finally, the zonal prediction maps of weathered bedrock water abundance in the southern wing of Ningtiaota coalfield under different weighting conditions are obtained (Fig 3, Fig 4 and Fig 5). According to the natural discontinuity method, the water abundance is divided into four grades: extremely weak (I), weak (II), medium (III) and strong (IV).

According to the characteristics of unit inflow of 41 pumping holes in this area, the unit inflow of borehole is divided into four categories, namely less than 0.01L/s·m, 0.01–0.1L/s·m, 0.1–1L/s·m and 1–5L/ s·m corresponds to the extremely weak (I), weak (II), medium (III) and strong (IV) in Figures 3, 4 and 5 respectively (Table 5).

In order to compare the advantages and disadvantages of three water abundance partition prediction methods based on improved AHP, entropy weight method and the coupling method, the water abundance level of the weathered bedrock pumping

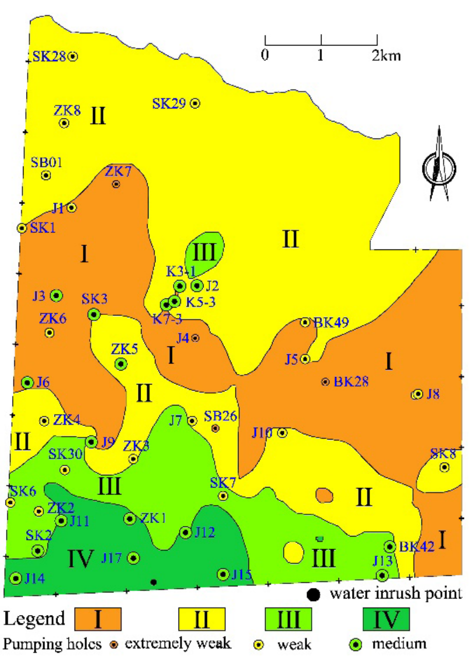

Fig.3 Water abundance prediction zoning map based on AHP

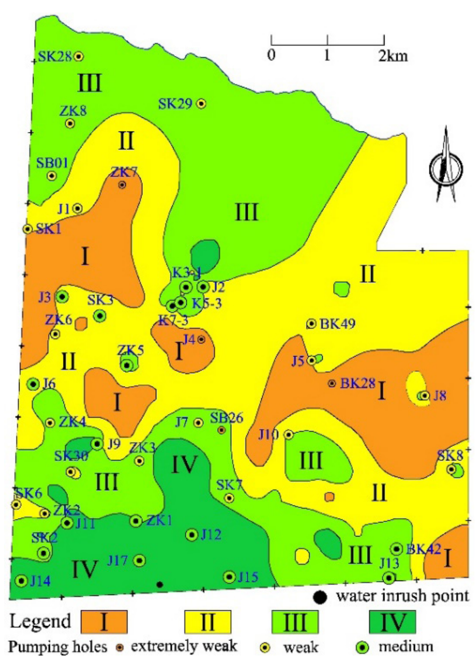

Fig.4 Water abundance partition prediction map based on the entropy weight method

Table 5 Correspondence of interrupt value and predicted partition

Interruption value (K)	<0.3996	0.3996–0.5449	0.5449–0.7029	0.7029–0.9304
Predictive partition	I	II	III	IV
Water abundance in pumping boreholes	extremely weak	weak	medium	medium

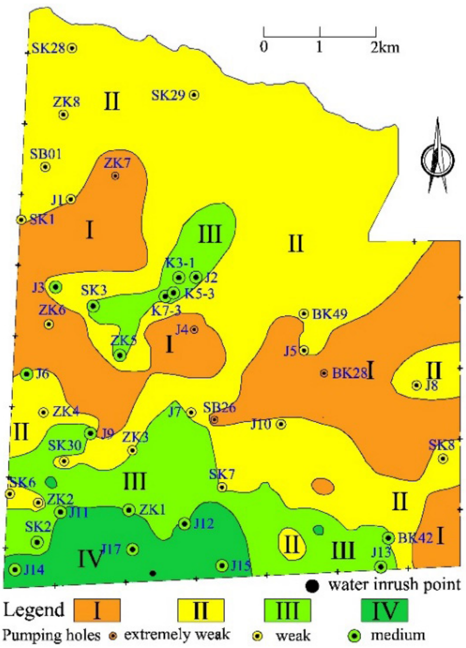

Fig.5 Water abundance partition prediction map based on improved AHP and the entropy weight method

borehole in the area is compared with forecast categories in Figures 3, 4 and 5. If they are consistent, they are considered to be the same, indicating that the prediction results are correct. By comparison, it is found that the water abundance classification predicted by the improved AHP method, the entropy method and the coupling method are 68.29%, 70.73% and 92.68% respectively. So the least squares method is the best method for water abundance zoning prediction.

Conclusion

The fissure aquifer of weathered bedrock in Jurassic coalfield in northern Shaanxi is the main water-filling aquifer in the mine. Its water abundance varies greatly in space. Four indexes can be used to predict the water richness of weathered bedrock aquifer, top-level index, lithologic association index, weathered index and core rate index.

The water abundance prediction method based on coupling of improved AHP and entropy weight method is a accurate method. It is more effective than the improved AHP and entropy weight method when used alone.

Acknowledgements

The research was supported by the National Natural Science Foundation of China (No. 41472234), Shaanxi Coal Industry Chemical Group Research Project of China (2017SMHKJ-C-23), Shaanxi Coal Industry Chemical Group Research Project of China (2018SMHKJ-C-31).

References

Chen DP, Jiang X. (2007) Research on coordinated development between socioeconomic system in Northern Shaanxi Energy & Chemical Industry Base. Environ. Sci. Manage. 09: 186-191.

Xi YT, Feng CL, Guo YH, Zheng SS. (2015) The theory and practice of conversion about specific capacity. Coal Geol. Explor. 43(01):48-51.

Dai GL, Yang T, Zhou Y, Niu C. (2016) Exploration of the water-abundant sandstone geological features of Ningtiaota Ida Nao Luo in Shenfu mining area. J. Saf. Environ. 16(04):144-148.

Wu Q, Fan ZL, Liu SQ, Zhang YW, Sun WJ. (2011) Water-richness evaluation method of water-filled aquifer based on the principle of information fusion with GIS: Water-richness index method. J. China Coal Soc. 36(07):1124-1128.

Hou EK, Tong RJ, Wang SJ, Feng J, Chen T. (2016) Prediction method for the water enrichment of weathered bedrock based on Fisher model in Northern Shaanxi Jurassic coalfield. J. China Coal Soc. 41(09):2312-2318.

Wang SQ, Niu JL, Liu Y, Du P, Zhang ZZ, Liang XS. (2014) Gushing Water Law and Its Control Factors Analysis of Jinjie Coal Mine. Saf. Coal Mines 45(02):145-147+150.

Guo JY, Zhang ZB, Sun QY. (2008) Study and Applications of Analytic Hierarchy Process. China Saf. Sci. J. (CSSJ)05:148-153.

Zhang HB, Luo YM, Zhao QG, Zhang GL, Huang MH. (2006) Hong Kong soil researches. Integrated evaluation of soil fertility quality based on the improved analytic hierarchy process. Acta Pedol. Sin. 04:577-583.

Ma R, Shi JS, Liu JC. (2011) Application of Entropy Weight and Stochastic Theory to Study the Heterogeneity Synthetic Index of Aquifer. Journal of Jilin University(Earth Science Edition) 41(05):1520-1528.

Wu DT, Li DF. (2004) Shortcomings of analytical hierarchy process and the path to improve the method. Journal of Beijing Normal University(Natural Science)02:264-268.

Huang DC, Zheng HR. (2003) Scale-Extending Method for Constructing Judgment Matrix in the Analytic Hierarchy Process. Syst. Eng. 01:105-109.

Pan HJ, Zhang DL. (2003) Problems and improvement methods of AHP method. Aero Weaponry 06:16-17.

Zhai LL, Zhang QH. (1994) Applied Decision Analysis. Tongji University Press.

Deng HL, Dai D, Li Sw. (2017) Comprehensive operation risk evaluation of overhead transmission line based on hierarchical analysis entropy weight method. Power System Protection and Control 45(01):28-34.

Kong SP, Zhang HR, Liao XP, Hong DP. (2018) Aerial Targets Threat Assessment Based on AHP and Entropy Weight Method. Tactical Missile Technology 01:79-84.