COMPARISON OF DARCY'S LAW AND FICK'S LAW OF DIFFUSION TO DETERMINE THE FIELD PARAMETERS RELATED TO METHANE GAS DRAINAGE IN COAL SEAMS

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ABSTRACT

This paper describes the applications of two mathematical models and numerical methods to describe gas drainage in coal seams. The mechanism of gas flow from coal is not very clear and can either follow Darcy's Law or Fick's Law of diffusion. At early stages where the gas pressure is high, Darcy's law applies whereas at later stages when gas pressure is stabilised, Fick's law of diffusion is applicable. Results for values of permeability are then compared to find correlation of these two methods.

INTRODUCTION

Parameters such as permeability, porosity, gas pressure, viscosity, etc influence adsorption/desorption characteristics of a coal seam. In most coal seams the contained gases comprise either methane or methane and carbon dioxide predominantly. This paper discusses the development of a mathematical model that describes the mechanism of methane gas flow in a coal bed during underground mining operations.

Problems related to the presence of methane gas in coal seams are on the increase in coal mining industries around the world because of increasing depths of mining and higher rates of production. Intensive mining at depth, in areas with high gas content results in higher gas emissions which create dangerous working conditions because of the inflammability of methane gas. Safety regulations require that the gas content of the mine atmosphere at places of work should not exceed 1.25% (N.S.W. Regulations Act 1984). Prediction of gas emission in mine workings requires development of methods for the measurement of gas pressure and gas flow rates. Techniques developed in the laboratory to determine permeability or diffusion parameters based on small samples are not reliable due to the scale effects which are very dominant in geological materials. As a result extrapolation of laboratory results to field conditions can lead to gross errors. Therefore it is essential to develop techniques that help to determine field parameters easily and reliably. The paper presents a method for the determination of field parameters for the flow of gas into mine workings.

In general, the mechanics of the flow of methane gas through a porous medium is very complex. This complexity is increased due to the fact that methane gas is present in the pores and fracture system (~10%) as occluded gas and in the internal surface area of coal matrix, as adsorbed gas (~90%)[1].

It is assumed that the large mass of the gas present as adsorbed gas must flow out or diffuse out of the coal matrix and that the laws of Darcy of diffusion are, therefore, most appropriate for the flow of gas through coal at a later stage.

RADIAL FLOW (Darcy Law)

This section presents a numerical method for describing the transient flow of water and gases radially inwards or outward through a porous medium. The pressure distribution as a function of time has been calculated for various ratios of reservoir radius to borehole radius. A simple model of predicting the well pressure at any time and at any radial distance for such an idealized field has been developed.

Immediately when a hole is drilled into the side of an opening, the pressure on the boundary wall of the borehole drops with respect to the insitu pressure causing radial flow into the borehole. In a two dimensional case of radial gas flow, the continuity equation is given below [1and 2] :

$$\frac{\partial}{\partial r}(\rho u) + \rho\frac{u}{r} = -\phi\frac{\partial \rho}{\partial t} \tag{1}$$

where ρ = density of gas

ϕ = porosity

r = radius

For an ideal gas, $\rho = \dfrac{Mp}{ZRT}$ (2)

where

p	=	pressure
M	=	molecular weight
R	=	gas constant
T	=	temperature
Z	=	compressibility factor

From Darcy's Law

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial r} \tag{3}$$

where K = permeability

μ = viscosity

Substituting Equation (2) and Equation (3) into Equation (1) it can be shown

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\phi\mu}{Kp}\frac{\partial p^2}{\partial t} \tag{4}$$

or, re-arranging terms Equation (4) can be re-written as

$$\frac{\partial p^2}{\partial t} = \frac{Kp}{\mu\phi r^2}\frac{\partial^2 p^2}{\partial(\ln r)^2} \tag{5}$$

Using dimensionless quantities

$$P = \frac{p}{p_o}, \; R = \frac{r}{r_o}, \; t_D = \frac{t}{\tau}$$

$$\text{where } \tau = \frac{\mu \phi r_o^2}{K p_o}$$

The equation (5) reduces to

$$\frac{\partial P^2}{\partial t_D} = \frac{1}{R^2} \frac{\partial^2 P^2}{\partial (\ln R)^2} \qquad (6)$$

EXAMPLE FOR CALCULATING RADIAL FLOW

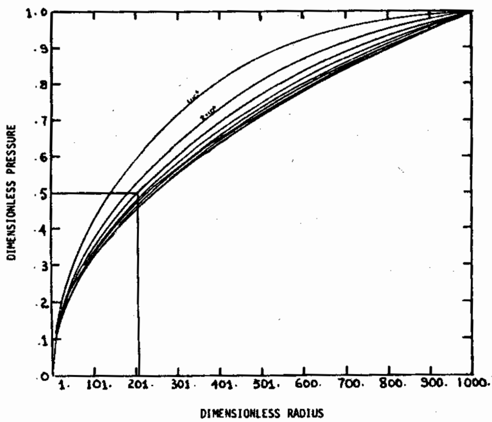

Figure 1 Prediction of gas pressure drop in coal seams against radial distance using Darcy's equation.

If it is required to determine the pressure drops at a distance 100 times the radius (r_o) of the hole, after a time of say, 1000 mins.

Let the values of various variables be as follows:

	μ	=	0.7cp
	ϕ	=	0.1
	p_o	=	100 atm
	r_o	=	12.5cm
	τ	=	1100secs
which gives	K	=	10^{-4} Darcy and 1 Darcy = 0.962 m/day
	t_D	=	550

Using Crank Nicholsan to equation (6), dimensionless pressure P is calculated in terms of dimensionless radius R. From Figure 1 projecting a line from the dimensionless radius $R = \frac{r}{r_o} = 200$ we get $P = \frac{p}{p_o} = 0.5$ at $t_D = 550$. Thus, the pressure is 0.5 of the original shut in pressure.

RADIAL FLOW (DIFFUSION EQUATION)

Flow through porous media in reservoirs may be treated as steady- state when conditions do not change with time . Pressure depletion of a gas field upon gas withdrawal is an unsteady- state phenonmenon. However, when flow has become stabilized steady state conditions prevail. The principal application of this method is the radial diffusion of gas outwards through a porous medium as governed by Fick's diffusion equation [4]. By solving the radial diffusion equation, loss of gas concentration at any radial distance and at any time can be predicted.

The radial diffusion equation [4] is given as follows:

$$D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) = \frac{\partial C}{\partial t} \tag{7}$$

where

C = concentration or quantity of gas in the coal medium
D = coefficient of Diffusion
$= \frac{Kp}{\mu\phi}$

K, p, μ and ϕ are described before.

The boundary conditions are:

$$\begin{array}{llll} C = C_o, & r = r_o, & t = 0 \\ C = 0, & r = r_o, & t > 0 \\ C = C_o, & r \to \infty & t \to \infty \end{array} \tag{8}$$

Using the dimensionless variables

$$\bar{C} = \frac{C}{C_0}, \quad R = \frac{r}{r_o}, \quad t_D = \frac{t}{\tau}, \quad \tau = \frac{\mu\phi r_0^2}{Kp_0}$$

the diffusion equation (7) and the boundary conditions (8) reduce to

$$\frac{\partial^2 \bar{C}}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{C}}{\partial R} = \frac{\partial \bar{C}}{\partial t_D} \tag{9}$$

and

$$\begin{array}{llll} \bar{C} = 1 & 0 \le R \le 1, & t_D \le 0 \\ \bar{C} = 0, & R = 1, & t_D > 0 \\ \bar{C} = 1, & R \to \infty, & t_D \to \infty \end{array} \tag{10}$$

Solution to the above equations (9) and (10) is given by

$$\bar{C} = \pi \sum_{n=1}^{\infty} e^{-\alpha_n^2 t_D} \frac{\rho_n U_o(\alpha_n R)}{1+\rho_n}$$

$$- \pi \sum_{n=1}^{\infty} e^{-\alpha_n^2 t_D} \frac{\rho_n^2 U_n(\alpha_n R)}{\rho_n^2 - 1} + \frac{\ln R}{\ln k} \tag{11}$$

where $\rho_n = \dfrac{J_o(\alpha_n)}{J_o(k\alpha_n)}$, $k = \dfrac{r_b}{r_0}$, J_o = Bessel function of first kind of zero.

and α is the root of the equation:

$$J_0(\alpha)\ Y_0(k\alpha) - Y_0(\alpha)\ J_0(k\alpha) = 0 \tag{12}$$

and r_b is the outer radius of the region.

The solution of equation (11) is represented graphically in Figure 2 for various values of dimensionless time t_D.

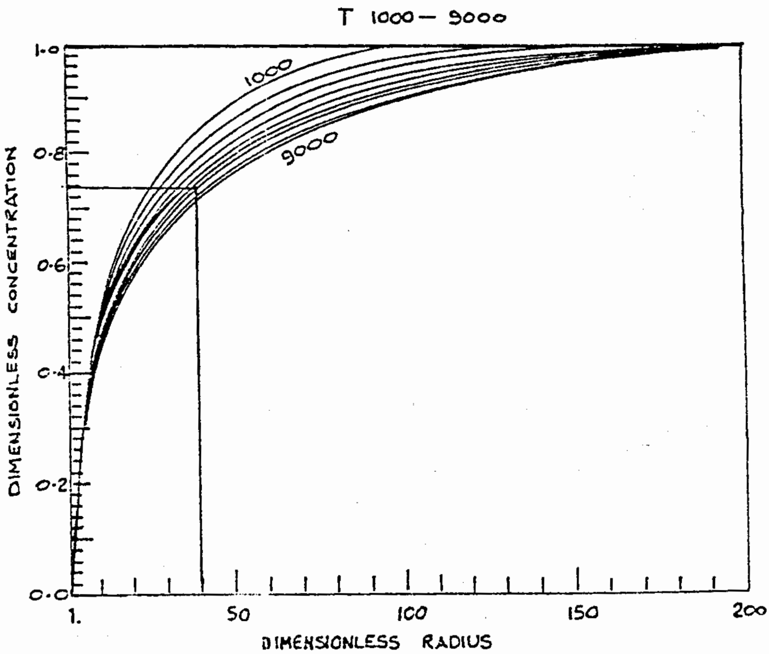

Figure 2 Prediction of gas concentration with radial distance using Fick's Law

From Figure 2 projecting a line from the dimensionless radius $R = r/r_0 = 40$.

We get $\bar{C} = C/C_0 = 0.74$, $t_D = 6000$. By varying the diffusion parameters, which in effect change the time parameter τ, it is possible to predict the methane gas concentration at any distance and time ahead of a mine face. So there is a close relationship between the concentration $\bar{C}$, radius of the borehole R, and the time parameter τ.

COMPARISON OF THE TWO METHODS

Using the relationship $\tau = \frac{\mu\phi r_0^2}{K\ p_0}$ for both Darcy's law and Diffusion method we can find a graphical relation between time and permeability K, assuming constant values, of, μ, ϕ, r_0 and p_0. The mathematical relation is of the form xy = constant, which is of the hyperbola form. From the graphs it is obvious that the two methods are very close to each other and the values of permeability can be calculated from either of the two graphs for any length of time.

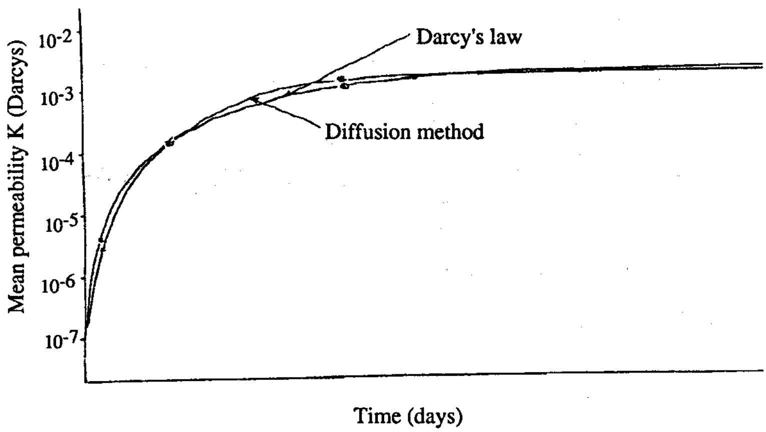

Figure 3 Comparison of Darcy's method and diffusion method for the calculation of permeability

It can be seen from the above results that there exists a close relation between these two methods.

APPLICATION OF DARCY'S MODEL TO MORE THAN ONE BOREHOLE

Differential equation for unsteady flow of gas and water is non-linear especially when there are numbers of boreholes. As a rule no analytical solution exists and a numerical technique for two phase flow is applied in this case.

Tests were conducted at a major underground colliery operating the Illawarra Coal measure [Ref.6]. Precautions were taken to stabilise the machine and accurately orient the direction of drilling. Packers were installed in the drilled holes and pressures were measured over a period of up to 300 hours to ensure that the system had stabilised and the maximum pressures had been achieved. In this period, the pressures slowly dropped due to natural drainage after the peak was achieved at different measuring points. A 24 m deep drainage hole of 100 mm diameter was constructed, and six 30 m deep parallel boreholes with pressure sensors were drilled along a line at distance 4,6,9,25,35 m from the drainage hole. In each borehole, pressure readings were taken at 0,10,20 and 30 m distances. Pressure readings obtained in this way are shown in Table 1.

Table 1
Observed Initial Borehole Gas Pressures

Distance into Borehole (m)	Distance of Borehole from Drainage Hole (m)						
	0	4	6	9	16	25	35
0	80*#	80*	80*	80*	80*	80*	80*
10	80#	-	-	93	96	99	108
20	80#	140	-	290	348	-	359
30	80#	854	854	860	878	892	887

*Pressures in mine entry. #Pressures in drainage hole
Note: Values are absolute pressures in kPa.

The planned layout of holes is given in Figure 4.

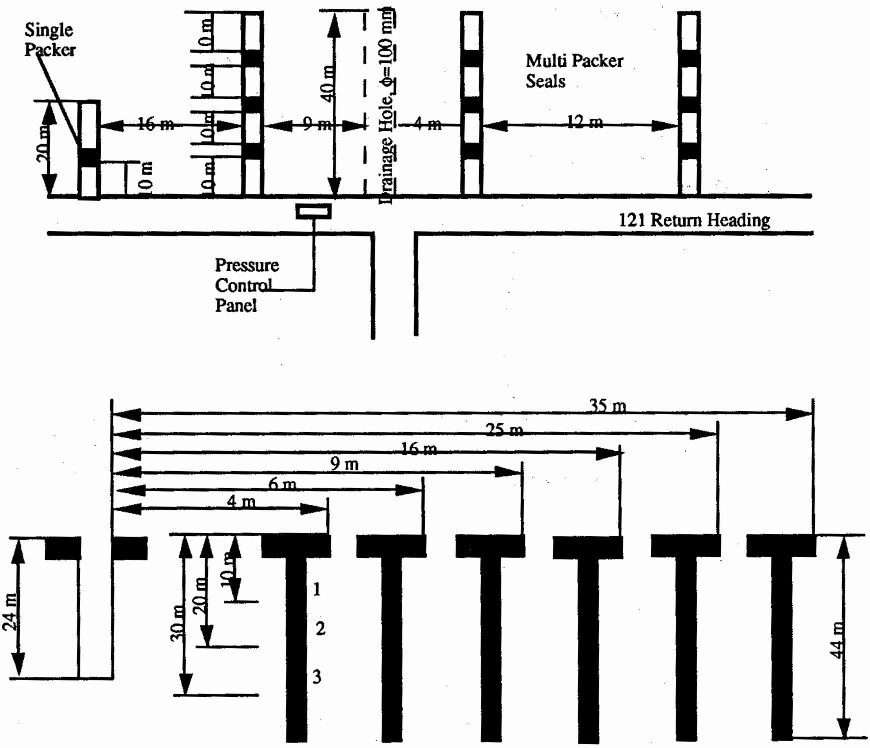

Figure 4 Test arrangement to determine drainage influence

The following data for the actual experimental condition sphere were used:

Average temperature in mine entry	22°
Water viscosity	1.0×20^{-3} Pa s
Gas viscosity	1.3×10^{-5} Pa s
Porosity	0.08
Absolute permeability	88 milli Darcys
Water density	1000 kg/m^3
Gas density	0.52 kg/m^3 at 80 kPa
Methane gas constant, R	518 J/kg.K
Gas-water relative permeability	Figure 5
Capillary pressure vs water saturation	Figure 6
Average pressure in mine entry	80 kPa

Using Figures 5 and 6 , values of gas relative permeability and capillary pressure with respect to water saturation are used in numerical model [7].

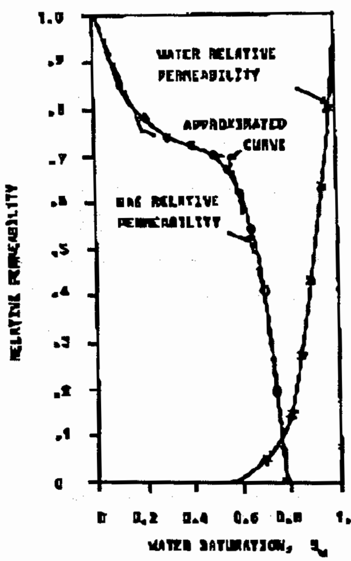

Figure 5 Relative permeability of gas and water vs water saturation

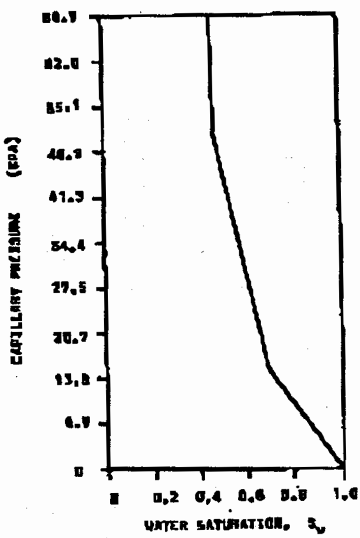

Figure 6 Capillary pressure vs water saturation

Pressures at all points in the coal seam were interpolated from Table 1 using a cubic spline curve.

Unfortunately, permeability in coal seams can vary widely, from 50 milli-Darcy to 1 Darcy. In particular, accurate values of permeability have not yet been determined for the particular coal seam under study ; however a value of 80 milli-Darcy has been suggested by engineers of the operating mine [Ref. 5]. In view of this recommendation, all calculations are based on permeability of 80 milli Darcy. The effect of assuming a different permeability (1 Darcy) was also studied using the numerical model incorporating two phase implicit finite difference technique.

The following boundary conditions at the borehole ($x = 0, t = 0$) are used:

p_g = 80 kPa, gas pressure
s_w = 0.763, water saturation
p_c = 12 kPa, capillary pressure
p_w = 68 kPa, water pressure

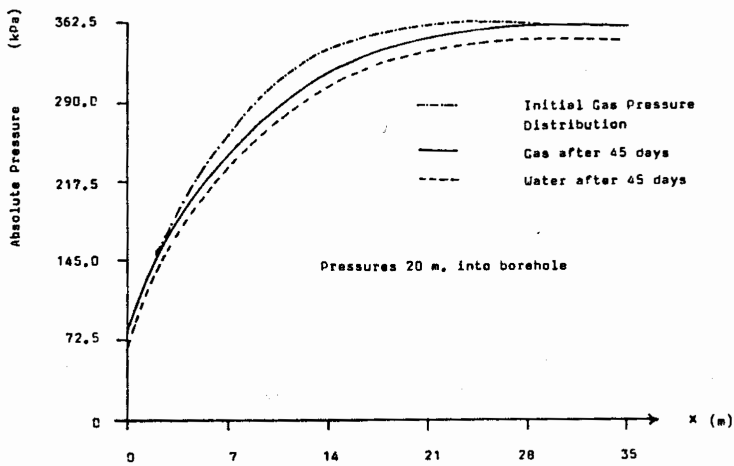

Figure 7 Calculated pressure prediction of gas and water for multiple boreholes using Darcy's Law (45 days)

RESULTS OF COMPUTER MODELLING

Figures 7 to 10 show pressure distributions of gas and water within the coal seam calculated at times 45 and 90 days respectively after drainage commences using the parameter values given above.

Obviously, pressures generally decrease as flow occurs towards the drainage borehole. At all points in the coal seam, pressures decrease with elapsed drainage time. This is more noticeable at points 30 m into the boreholes where the pressure gradient is steeper.

For distances in excess of 45 m from the drainage hole, pressure of both gas and water remain close to initial values and do not decrease with time.

Figure 11 shows flow rates of gas and water per metre of borehole length, calculated over a 90 days period. Flow rates of water remain essentially constant with time and this agrees with experimental measurements taken at nearby operating coal mine. The flow rate of methane gas decreases slightly with increasing time as the gas content of the coal seam is depleted. The maximum value of 2.4 litres/minute/metre length of borehole agrees reasonably well with the experimental values of 1.96 litres/minute/metre observed at after mines operating the same seam.

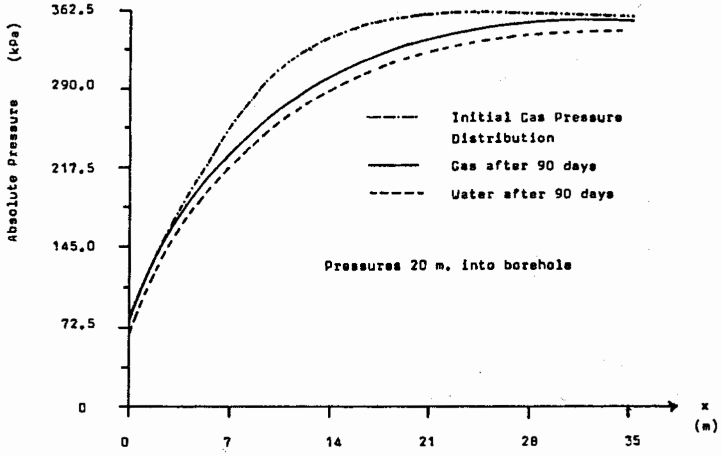

Figure 8 Calculated pressure Prediction of gas and water for multiple boreholes using Darcy's Law (90 days)

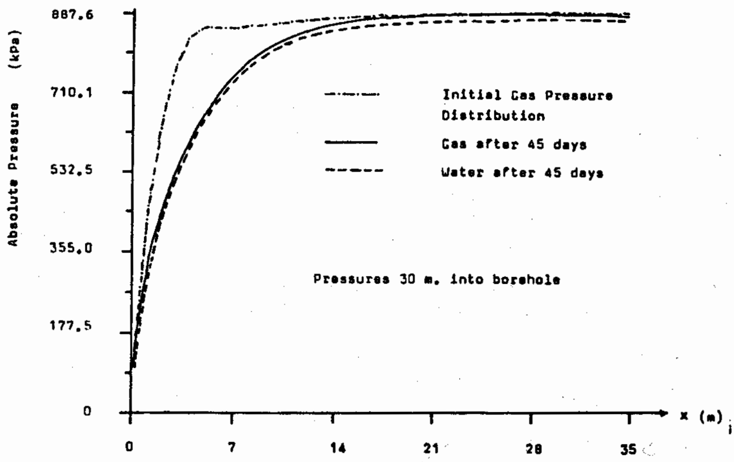

Figure 9 Calculated prediction of gas and water pressure 45 days after drainage commences (20m borehole)

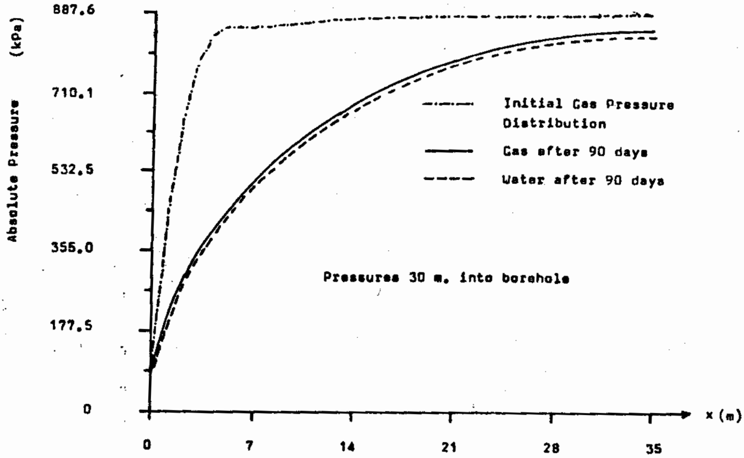

Figure 10 Calculated gas and water pressure distributions 90 days after drainage commences (30m borehole)

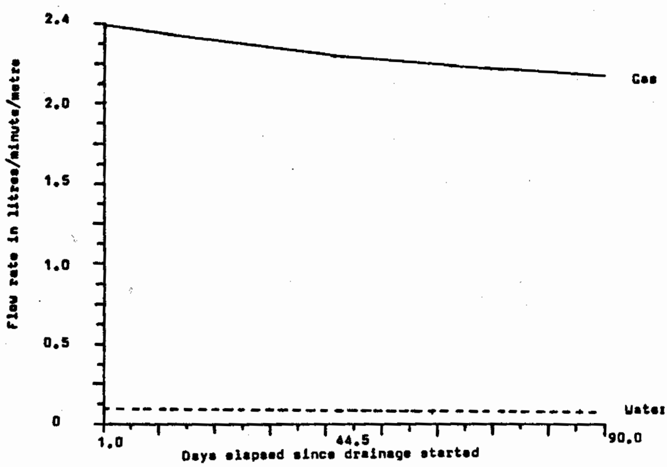

Figure 11 Calculated flow rates of gas and water per metre of borehole length (90 day period)

CONCLUSIONS

The first model is based on solution of the continuity equation and Darcy equation, applied to each phase. The resulting coupled equations are solved in a finite difference formulation.

The second model based on diffusion method gives analytical solution which is then compared with that of the first model as shown in Figure 3.

The computer model has been applied to experimental measurements taken at an operating coal mine in the Illawarra coal measures, NSW, Australia. Agreement between predicted and experimental pressure distributions and flow rates is generally good.

It has been shown that for mathematical modelling using either Darcy's law or Fick's law is suitable for gas/water drainage in coal seams. In the future Fick's law of diffusion will be applied for multiple boreholes.

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