# COMPUTER AIDED PREDICTION OF LAND SUBSIDENCE DUE TO GROUNDWATER WITHDRAWAL

by

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## ABSTRACT

Stress relieving accompanied by a decrease in pressure and caused by concentrated water withdrawal from confined aquifers causes compression of overlaying strata. The summarized effect in the mainly compressible strata with low permeability of the water bearing system appears as land subsidence, which changes by areas and damages the technical facilities on or near the surface. This paper presents a model for prediction of land subsidence due to groundwater withdrawal and deals with its application for the prediction of land subsidence to be expected on the territory of the open-pit mine called Thorez.

## INTRODUCTION

According to international literature, solutions - used internationally in the prediction of land subsidence - can be divided into 3 groups:

- The majority of specialists apply empirical solutions and - confirmed by dozens of practical examples - consider them to be of satisfactory accuracy. In most empirical solutions only time is considered an independent variable and the constants which can be found in land subsidence - time functions are determined from the data of in situ investigations (e.g. Buissman, 1936; Vyalov, 1986; Poland, 1969; Kodandaramaswamy-Narasimhan, 1980).

- In semi-empirical solutions in addition to time the cause of the settling effect also appears in the form of the application of a value of some physico-mechanical parameter in a theoretical function. The physico-mechanical parameters (e.g. compressibility factor, index of consolidation, modulus of elasticity, Poisson's coefficient) are measured in laboratory or in situ. Semi-empirical solutions were used e.g. by Suklje (1972) and by Vyalov (1986).

- The theoretical solutions explain the cause of subsidence by the seepage from the impermeable layers of the water bearing system (Terzaghi' s principle), which is caused by stress relieving in the water bearing layers (principle of effective stress). Rheological rock behaviour is either not taken into account, or it is taken into account by plasticity law, by laws of ideal rheological materials, or by government laws of physico-mechanics (e.g. Taylor, 1940; Fukuo,1969; Somosvári, 1988.).



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The theoretical functions are solved in connected systems by specialists who investigate the stress relieving process in connection with the compaction caused by increase in effective stress and work with only one model to analyse the predictable amount of land subsidence (e.g. Florin, 1959; Duncan-Chang, 1970; Richter, 1979; Kavazanjian-Mitchell, 1980; Worsak-Chau, 1990).

Uncoupled systems are used by the specialists who take into consiredation the amount of the depression referring to a particular place as known (measured or predicted by some model) at the beginning of every time-step, and putting it into a consolidation model of some type they calculate the amount of compaction by using this value (e.g. Mironyenko-Sesztakov, 1974; Vyalov, 1986; Kedwell, 1984; Booker-Small, 1986).

In the system of the models described, our model is a semi-empirical one which uses the time-legs of depression and land subsidence to take into consideration the effect of preconsolidation and to approximate the creeping process following the seepage from the impermeable layers.

## PHISICAL MOODEL

Our physical model used for prediction of land subsidence is a double-level one. The first level models an isotherm, transient flow taking place in compressible porous media. The space is divided into permeable and impermeable layers (see Figure 1) with the following assumptions:

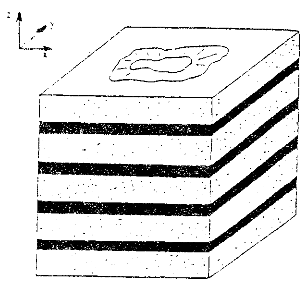

Figure 1.
Physical model of
hydrodinamic flow

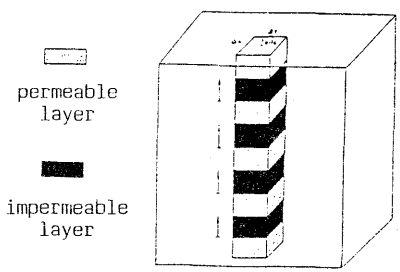


Figure 2.
Physical model of
land subsidences

- the direction of the flow is horizontal in the permeable layers and can be described by a 2-dimensional model;

- the compressibility of the permeable layers is negligible as compared with that of the impermeable layers;

- the direction of the seepage in the impermeable layers is vertical, and the seepage proceeds at the same rate as their compaction;

- the effect of the water movement in the impermeable layer on the flow in the permeable layers is negligible;

- the horizontal water movement in the impermeable layers is negligible.



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The second level of our model is a one-dimensional consolidation model which is coupled with the first level by the principle of effective stress. The effect of preconsolidation is taken into account by a threshold value of the hydraulic gradient $(I_o)$ which is determined from a series of in situ observations. It means using the modified Darcy-law.

The second level of our model describes the subsidence caused by the compaction of the impermeable layers effected by the preconsolidation stress, in the vicinity of a given point $P(x_o, y_o)$(see Figura 2.), the two systems are connected through the permeable layers. The theoretical principle of the second level of our model is the classic one-dimensional consolidation theory of elastic materials (Terzaghi's principle).

## MATHEMATICAL MODEL

The isotherm single-phase flow of the compressible fluid moving in the macroscopic section of the permeable layer of the hydrogeological model (see Figure 1.) can generally be described by the conservation law:

$$\frac{\partial(\rho\phi)}{\partial t} = -div(\rho v) + q \qquad (1)$$

ρ - density of the water;
$\phi$ - porosity of the permeable layers;
$v$ - rate of the water movement;
$q$ - water - mass intensity of the springs and sinks.

Supposing that the porous system is completely filled with water and that the Drarcy-law is valid in the permeable layers $v$ can be written as:

$$v = -k \cdot grad(p + \rho \cdot g \cdot h) \qquad (2)$$

where
$k$ - permeability coefficient of the permeable layers;
$p$ - pressure of the pore-fluid;
$g$ - acceleration coefficient;
$h$ - elevation potential.

According to the principle of effective stress:

$$\sigma_z^t = \sigma_z^e + p \quad , \qquad (3)$$



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where

$\sigma_z^t$ - total stress (in vertical direction);

$\sigma_z^e$ - effective stress (in vertical direction).

As $\sigma_z^t = const., \partial_z^e = -\partial p$ and $d\sigma_z^e = -dp$, furthermore the compressibility of the pore-fluid is:

$$C_v = \frac{1}{\rho}\frac{d\rho}{dp}, \tag{4}$$

vertical compressibility of the porous rock is:

$$C_k = \frac{1}{\phi}\frac{d\phi}{dp}, \tag{5}$$

and the total compressibility of the system is:

$$C_t = C_v + C_k \tag{6}$$

The general form of Equation 1 is:

$$\rho\phi C_t \frac{\partial p}{\partial t} = div(\rho.k.gradp) + q, \tag{7}$$

where $\phi, \rho, C_t, k$ and $p$ are average values referring to the underlaying $h_a$ and the overlaying $h_f$ strata.

Due to the effect of preconsolidation the isotherm single-phase water movement can be described by the modified Darcy-law in the macroscopic section of the impermeable layer of the hydrogeological model:

$$v_z = K \cdot grad\bar{p} \tag{8}$$

where

$$\bar{p} = p - p_o;\ grad\bar{p} = \frac{dp}{dz} - p.g.I_o \tag{9}$$

$I_o$ - threshold value of the hydraulic gradient referring to the beginning of water seepage from the impermeable layer.

$K$ - permeability coefficient of the impermeable layer.

Based on the foregoing, the mathematical model of the one-dimensional consolidation can be written as:



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$$\frac{\partial \overline{p}}{\partial t} = c_v \frac{\partial^2 \overline{p}}{\partial z^2} \quad , \tag{10}$$

where

$$c_v = \frac{K(1+e)}{\rho \cdot g \cdot a_v} \, , \qquad a_v = \frac{de}{dp} \quad . \tag{11}$$

In this model the coefficient of consolidation $c_v$, void ratio $e$, compressibility $a_v$, permeability coefficient $K$ depend on the pore pressure directly or - through the change of void ratio $e$ - indirectly, depending on the chosen empirical model.

The initial and boundary conditions referring to the impermeable layer are the empirically corrected pore pressures of permeable layers, so the movement of the $k$-th impermeable layer of thickness $H^k$ is:

$$\Delta S^k(t) = \int_o^{H^k} m_v \left[ \overline{p}^k(z,o) - \overline{p}^k(z,t) \right] dz \tag{12}$$

The total surface movement is the sum of the movements of the individual layers whose number is $N$:

$$S(t) = \sum_{k=1}^{N} \Delta S^k \tag{13}$$

where

$$m_v = \frac{a_v}{1+e_o} \tag{14}$$

The primary consolidation of the individual impermeable layers is completed after the consolidation time referring to the maximum depression. Considering the process of consolidation to be completed at the amount of dissipation of 97 percent the time of dissipation $(t_o)$ is determined for the value of time factor $T$ referring to a given amount of compaction and read from a diagram. For $t_o$ the amount of consolidation of the $k$-th layer $\left[\Delta S^k(t_o)\right]$ is determined using which the amount of secondary consolidation derived from the rheological behaviour of the rocks can be approximated by the following function:

$$\Delta S^k(t)\Big|_{t > t_o} = \Delta S^k(t_o) + \frac{dS^k(t)}{dt}\Big|_{t=t_o} \cdot \ln t \tag{15}$$



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## NUMERICAL MODELL

The equation of the two-dimensional flow in the permeable layer can be approximated by the well-know method of finite differences, and can be solved by using the predictor corrector formula supported at two points. The results of the solutions are the boundary conditions of the pressure of impermeable layers.

The equation of the one-dimensional flow in the impermeable layer (10) can be solved by using the numerical model as follows:

$$\frac{\partial \overline{p}_i}{\partial t} \approx C_v \left( \frac{\overline{p}_{i+1} - \overline{p}_i}{z_{i+1} - z_i} - \frac{\overline{p}_i - \overline{p}_{i-1}}{z_i - z_{i-1}} \right) / \left( \frac{z_{i+1} - z_{i-1}}{2} \right) \tag{16}$$

where $\overline{p} = \overline{p}(z_i, t)$ and $z_i$ indicates the distance of the $i$-th finite difference element from the bottom of the layer.

After writing Equation 16 for every point $z_v$, we obtain an ordinary linear differential equation-system and using the following transcriptions.

$$\overline{p}(t) = \left[ \overline{p}_o(t), \overline{p}_1(t), \ldots \overline{p}_{N-1}(t), \overline{p}_N(t) \right]$$

$$F(\overline{p}(t)\ ) \left[ \ldots, C_v \left( \frac{\overline{p}_{i+1} - \overline{p}_i}{z_{i+1} - z_i} - \frac{\overline{p}_i - \overline{p}_{i-1}}{z_i - z_{i-1}} \right) \right] / \left( \frac{z_{i+1} - z_{i-1}}{2} \right), \ldots \ ] \tag{17}$$

Equation 16 can be written as

$$\frac{\partial \overline{p}(t)}{\partial t} = F(\overline{p}(t)\ ) \tag{18}$$

Equation-system 18 together with the pressure of permeable layers as boundary conditions is a correctly set task which can be solved by using the predictor corrector formula supported at two points. The rock movement described by Equation 12 can be approximated by the following integral formula:

$$\Delta S^k(t) \approx \sum_{i=0}^{N-1} m_v (z_{i+1} - z_i) \left( \overline{p}^k (z_{i+1}, o) - \left( \overline{p}(z_{i+1}, t) + \overline{p}(z_i, o) - \overline{p}(z_i, t\ ) \right) \right) / 2 \tag{19}$$

Function 15 can be determined by using functions 13 and 16, its summation referring to each layer gives the total land subsidence.

The land subsidence due to mining groundwater withdrawal from the open-pit mine Thorez predicted for the year 2000 can be seen in Figure 3.



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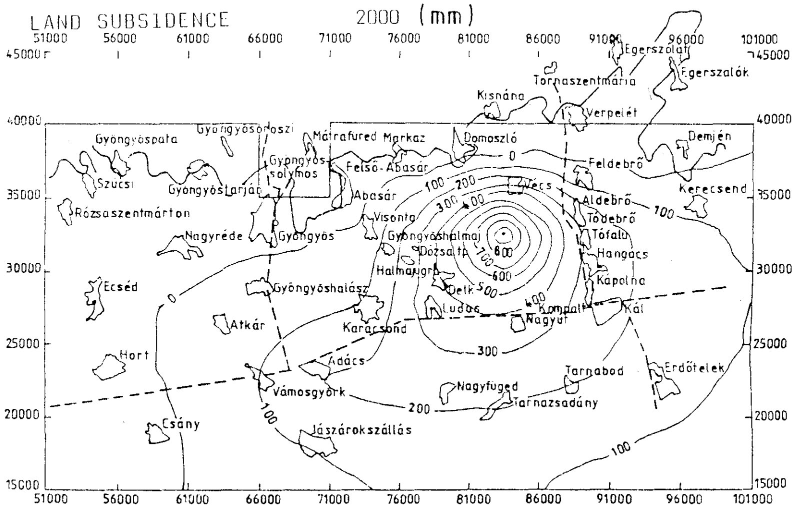


Figure 3.

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